5.6 Triangle Inequality Theorem

Draw a triangle ABC with the lengths 3, 5, 9 centimeters. Since we all have the same lengths, our triangles should be congruent by SSS. (Let the bottom AC length be 9 cm)

You cannot draw this triangle because the **Triangle Inequality Theorem** requires certain parameters.

A) The two smaller sides of a triangle when added together will be bigger than the third side. This addition will be the maximum. Likewise, the two smaller sides when subtracted from each other gives the minimum size.

So in our case, **min < x < max** $5 - 3 < x < 5 + 3$

$$2 < x < 8$$

Notice it is not equal, that means the length of the third side won’t ever get to 8 cm nor be as small as 2 cm.

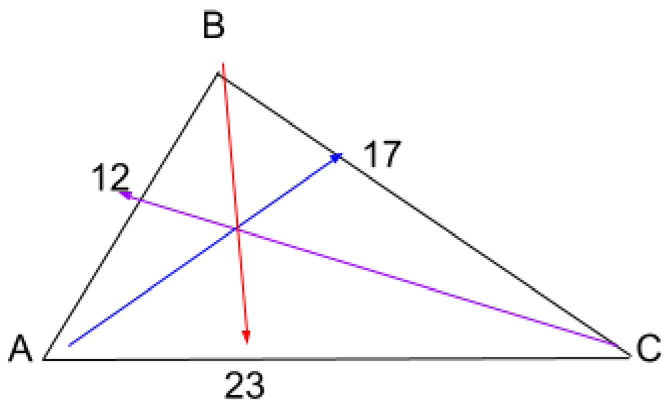
Examples: What is the range of values that the **third** side could be?

1. 8, 13   13 - 8 = **5 < x < 21** = 8 + 13
2. 14, 26   26 - 14 = **12 < x < 40** = 26 + 14
3. 8,15   15 - 8 = **7 < x < 23** = 15 + 8

B) So if we add to the above concept, when given three side lengths of a triangle, we can determine "yes/no the side lengths make a legal triangle or not.

1) 4, 9, 12 yes these side lengths make a legal triangle since 4 + 9 > 12
2) 6, 8, 14 Not a legal triangle, b/c 6 + 8 is not greater than 14
3) 5, 18, 9 Not a legal triangle b/c 5 + 9 is not > 18
4) 6, 6, 11 yes a legal triangle since 6 + 6 > 11

C) The third observation of the Triangle Inequality Theorem is it will tell you the order of the angles from small to medium to large. Need to look at the angle and look across from the angle to the opposite side.

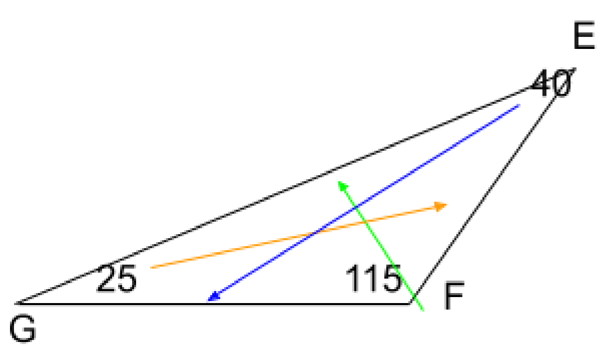


In the example above, 12 < 17 < 23 therefore

m<C < m<A < m<B

An easy way to remember this “Bigger sides produce bigger angles”

C) IF we reverse the information so that we are given the angles then the very same concept will work for the ordering of the sides.



25 < 40 < 115 so

EF < GF < GE

Again, an easy way to remember, Bigger angles produce bigger sides

So to finalize our discussion today, The Triangle Inequality Theorem tells us:

1) NO arbitrary side lengths
2) Range of values
3) Order the angles or sides