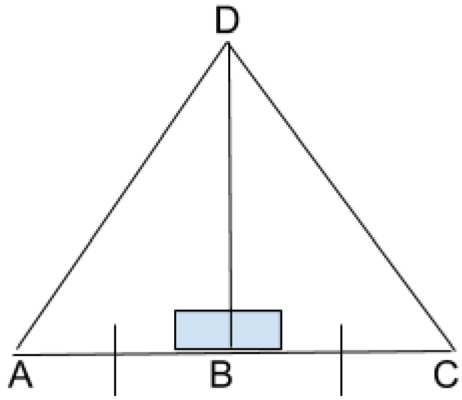
Perpendicular and Angle Bisector Theorems

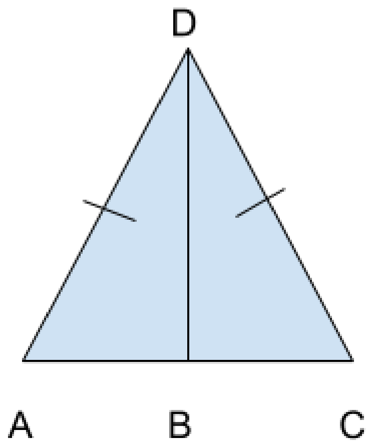
Perpendicular Bisector Theorem



Given: $\overline{AB} \cong \overline{CB}$ *and* $\overline{DB} \perp \overline{AC}$
Prove: $\overline{AD} \cong \overline{DC}$

1. $\overline{AB} \cong \overline{CB}$ *and* $\overline{DB} \perp \overline{AC}$	1. Given
2. $< DBA \cong < DBC$	2. $\perp \rightarrow$ *right angles* $\rightarrow \cong$
3. $\overline{DB} \cong \overline{DB}$	2. Reflexive property
4. $\Delta ABD \cong \Delta CBD$	3. SAS
5. $\overline{AD} \cong \overline{CD}$	4. CPCTC

Converse of the Perpendicular Bisector Theorem



Given: $\overline{AD} \simeq \overline{CD}$

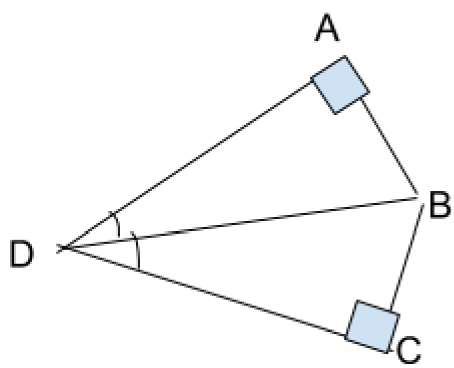
Prove: $\overline{AB} \simeq \overline{CB}$ *and* $\overline{DB} \perp \overline{AC}$

1. $\overline{AD} \simeq \overline{CD}$	1. Given
2. ΔADC *is isosceles*	2. Def of Isosceles triangle
3. $\overline{AB} \simeq \overline{CB}$ *and* $\overline{DB} \perp \overline{AC}$	3. Isosceles Triangle Theorems

What do these theorems mean?

a) No matter where point D is located on the perpendicular bisector then it is the same distance from the endpoints A, C
b) Point D is always on the Perpendicular Bisector

Angle Bisector Theorem

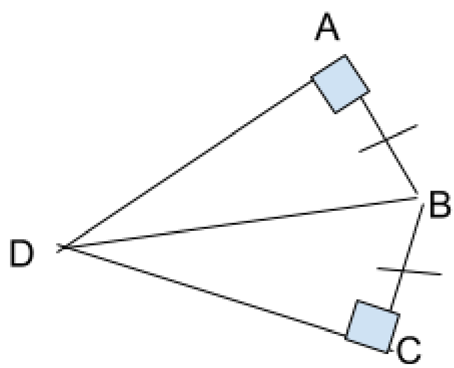


Given: $< ADB \cong < CDB$
Prove: $\overline{BA} \cong \overline{BC}$

1. $< ADB \cong < CDB$	1. Given
2. Draw $\overline{BC}$ and $\overline{BA}$ so that they are perpendicular to $\overline{DC}$ and $\overline{DA}$	2. Construction
3. $< BCD \cong < BAD$	3. $\perp \rightarrow$ right angles $\rightarrow \cong$
4. $\overline{DB} \cong \overline{DB}$	4. Reflexive property

5. $\Delta BDA \simeq BCD$	5. AAS
6. $\overline{BA} \simeq \overline{BC}$	6.CPCTC

Converse of the Angle Bisector Theorem



Prove: $< BDC \simeq< BDA$

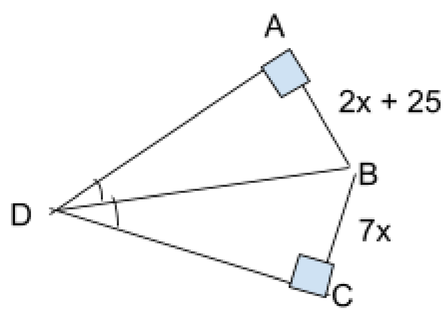
1. $\overline{BA} \simeq \overline{BC}$ *and* $\overline{BC} \perp \overline{BA}$	1. Given
2. $< C \simeq< A$	2. $\perp \rightarrow$ *right angles* $\rightarrow\simeq$
3. $\overline{BD} \simeq \overline{BD}$	3. Reflexive Property
4. $\Delta BCD \simeq \Delta BAD$	4. HL
5. $< BDC \simeq< BDA$	5. CPCTC

What do these theorems mean?

a) No matter where you are at on the angle bisector you are equidistant from the sides of the angle
b) If an interior point is equidistant from the sides of an angle then it lies on the angle bisector.

Examples:

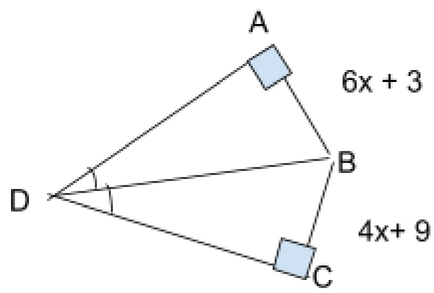
1. What is the length of BC?



$2x + 25 = 7x$
$25 = 5x$
$5 = x$

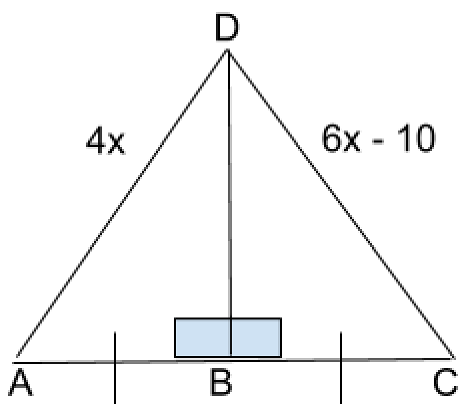
$BC = 7(5) = 35 = AB = 2(5) + 25$

2. What is the length of BA?



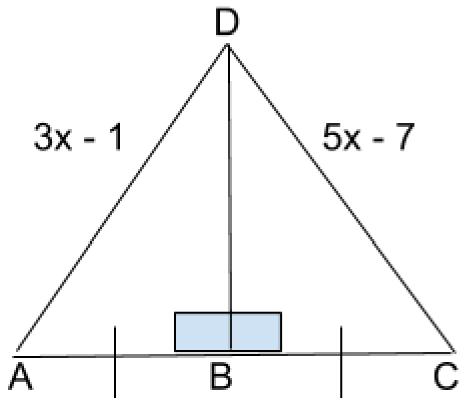
6x + 3 = 4x + 9
2x = 6
X = 3
AB = 6(3) +3 = 21 = BC = 4(3) + 9

3. What is the length of AD?



4x = 6x - 10
-2x = -10
X = 5
AD = 4(5) = 20 = DC = 6(5) - 10

4. What is the length CD?



3x - 1 = 5x - 7
-2x = -6
X = 3
DA = 3(3) - 1 = 8 = DC = 5(3) - 7

Reminder topic from chapter 3: Write an equation in **point-slope form** $(y - y_1) = m(x - x_1)$ for the perpendicular bisector to the given segment.

1. M(-5, 4) N(1,-2)

Steps: a) Find the slope of the **segment**: $\frac{4--2}{-5-1} = \frac{6}{-6} = -1$

What is the perpendicular slope? 1

b) Find the midpoint of the **segment**.

M $\left(\frac{-5+1}{2}, \frac{4+-2}{2}\right) = (-2, 1)$

c) Put steps 1 and 2 together to write the equation.
Simplify if necessary.

$(y - y_1) = m(x - x_1)$
Y - 1 = 1 (x - - 2)
Y - 1 = 1(x + 2)

2. U(2, -6), V(4,0)

Slope: m = $\frac{-6-0}{2-4} = \frac{-6}{-2} = 3$

So the perpendicular slope is -1/3

Midpoint: $\left(\frac{2 + 4}{2}, \frac{-6 + 0}{2}\right) = (3, -3)$

Equation: $y + 3 = -\frac{1}{3}(x - 3)$