Isosceles and Equilateral Triangle Theorem Notes


Vocabulary: < A: called the vertex angle, the congruent sides $A B$ and $A C$ are called the legs of the isosceles triangle and Side BC is called the base of the isosceles triangle. Angles B and C are the base angles.

There are two theorems about Isosceles Triangles and both are related but each has a separate proof. For simplicity to remember the theorems the following biconditional statement can be used to remember both.


A triangle is isosceles if and only if the base angles are congruent.

Proof that the base angles of an isosceles triangle are congruent. (Isosceles Triangle Theorem)


Given: $\overline{A B} \simeq \overline{A C}$
Prove: $<B \simeq<C$

| 1. $\overline{A B} \simeq \overline{A C}$ | 1. Given |
| :--- | :--- |
| 2. Draw $\overline{A D}$ such that | 2. Construction |
| $\overline{A D}$ bisects $<B A C$ |  |
| 3. $<D A B \simeq<D A C$ | 3. Def of bisect |
| 4. $\overline{A D} \simeq \overline{A D}$ | 4. Reflexive Prop |
| $5 . \Delta D A B \simeq \triangle D A C$ | 5. SAS |
| $6 .<B \simeq<C$ | 6. CPCTC |

Proof of the Converse of the Isosceles Triangle Theorem (If the base angles are congruent the sides are congruent)


Given: $<B \simeq<C$
Prove: $\overline{A B} \simeq \overline{A C}$

| 1. $<B \simeq<C$ | 1. Given |
| :--- | :--- |
| 2. Draw $\overline{A D}$ such that <br> $A D$ <br> bisects $<\mathrm{BAC}$ | 2. Construction |
| 3. $<D A B \simeq<D A C$ | 3. Def of bisect |
| 4. $\overline{A D} \simeq \overline{A D}$ | 4. Reflexive prop |
| 5. $\Delta B A D \simeq \triangle C A D$ | 5. AAS |
| 6. $\overline{A B} \simeq \overline{A C}$ | 6. CPCTC |

From these two theorems, what is also true?
a)Point $D$ is a Midpoint.
b) $<$ ADB and $<$ ADC are Right angles.

So practical questions:

| 1. |
| :--- | :--- |


| $12 x=48$ |
| :--- | :--- |
| $X=4$ |
| $9 * 4-14=22=R O$ |
| $-3 * 4+34=R S$ |


|  |  |
| :---: | :---: |
| 5. Suppose you have an exterior angle of an isosceles triangle that measures 130 degrees. What could be the interior angles? | 6. Suppose the perimeter of an isosceles triangle is 63 with the base length is $x$ and one of the legs is $5 x-7$. Solve for $x$ and what is the length of each side? |
|  |  |
|  | $\begin{aligned} & X+5 x-7+5 x-7=63 \\ & 11 x-14=63 \\ & 11 x=77 \\ & X=7 \end{aligned}$ |
| $\begin{aligned} & 180-130=50=x \\ & 50+50+y=180 \\ & 100+y=180 \\ & Y=80 \end{aligned}$ | $\begin{aligned} & \text { Base }=7 \\ & 5(7)-7=28=\text { legs } \end{aligned}$ |


| $50,50,80$ |  |
| :--- | :--- |
| or |  |
|  |  |

